

Coulomb Branch Gluon Scattering via AdS/CFT

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Motivation

- Alday and Maldacena give a prescription for computing the color ordered $gg \rightarrow gg$
 - such scattering amplitudes are not “IR safe” observables.
 - however play a role in hard scattering (hadronization is slow)
 - AdS/CFT provides a method for computing these quantities at strong coupling

Overview

Alday Maldacena
prescription

Coulomb branch
geometry

What can we
calculate?

Concluding
Remarks

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 - however play a role in hard scattering (hadronization is slow)
 - AdS/CFT provides a method for computing these quantities at strong coupling
- “Real world” physics: Spontaneous symmetry breaking
 - the “expected Higgs” $SU(2) \times U(1) \rightarrow U(1)$
 - GUT models: $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

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- An easy model to study via AdS/CFT is therefore $\mathcal{N} = 4$ SYM theory with $SU(n_1 + n_2)$ gauge group spontaneously broken to $SU(n_1) \times SU(n_2) \times U(1)$.

Plan of Talk

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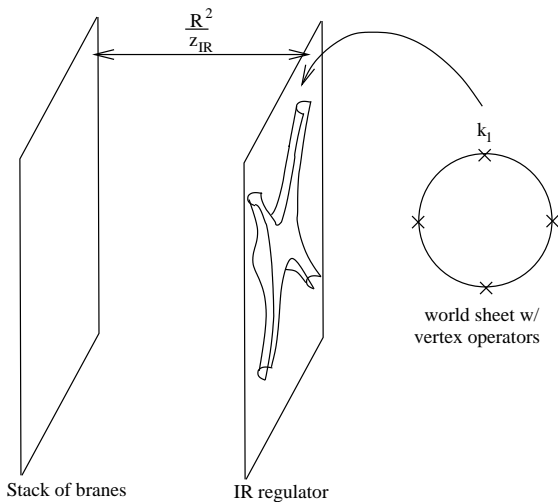
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- Alday-Maldacena prescription.
- The modification we examine: Coulomb branch of the theory.
- What can we compute? A simple example: massive “W”s in loops as corrections.

Review

- In AM, they introduce an IR regulator brane on which to have the open string end with 4 momenta k_i .



- Recall that we wish to fix the momentum conjugate to x^μ . However due to metric factors, the proper momentum of the string

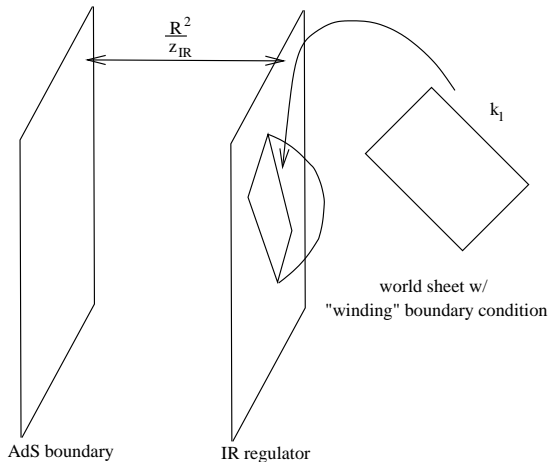
$$k_{pr} = \frac{z_{IR}}{R} k_{\text{conj}}$$

- So, when we remove the cutoff, we are talking about high proper momentum.
- Upshot of flat space calculation is that such a limit is dominated by the saddle point i.e. *classical* solutions.

- Final crucial ingredient: we need to fix momenta on the boundary of the classical world sheet.
- The boundary conditions are easier to impose after T-duality.
- Intuitively: T-duality maps “winding \leftrightarrow momentum”
- Nothing is compact: continuum momentum goes to continuum extension
- Metric:

$$\begin{aligned} ds^2 &= \frac{r^2}{R^2} (dx_\mu dx^\mu) + R^2 \frac{dr^2}{r^2} \\ &\rightarrow \frac{R^2}{r^2} (dx_\mu dx^\mu) + R^2 \frac{dr^2}{r^2} \end{aligned}$$

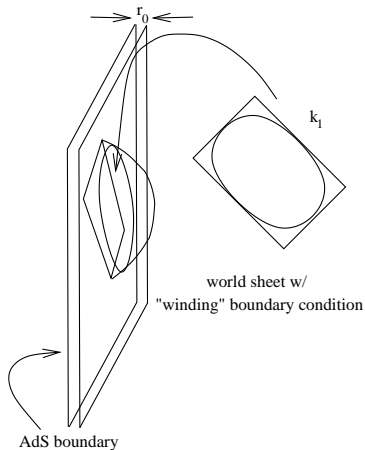
- T dualizing the geometry



- The AM prescription is now that the color ordered planar amplitude is

$$\mathcal{A}(k_1 \dots k_4) = e^{-S(k_1 \dots k_4)}$$

- Actual IR regularization impose



- In general, we will be examining geometries of the following kind

$$ds^2 = H^{-1/2} dx_\mu dx^\mu + H^{1/2} dz_m dz^m,$$
$$H = R^4 \left(\frac{n_1}{(\vec{r})^4} + \frac{n_2}{(\vec{r} - \vec{a}_0)^4} \right).$$

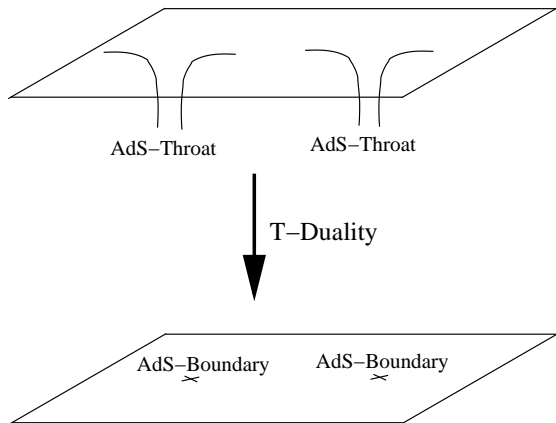
- Work at “north pole.”
- T-duality keeps $H(r)$ intact!

- T-duality: flip power of H in front of $dx_\mu dx^\mu$ and refer to the T-dual coordinates as y^μ .

$$ds^2 = H^{1/2}(r) dy_\mu dy^\mu + H^{1/2}(r) dr^2,$$
$$H = R^4 \left(\frac{n_1}{(r)^4} + \frac{n_2}{(r - a_0)^4} \right).$$

- Two AdS boundary regions.

T-duality action



- We make the Ansatz

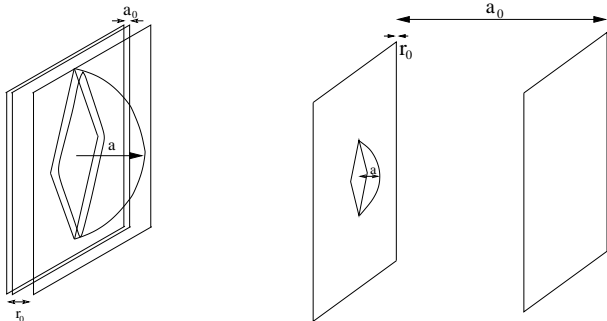
$$y_0(y_1, y_2), \quad r(y_1, y_2), \quad y_3 = 0$$

which gives the worldsheet action as

$$S = \frac{1}{2\pi\alpha'} \int dy_1 dy_2 H(r)^{\frac{1}{2}} \times \\ \sqrt{1 - (\partial_i y_0)^2 + (\partial_i r)^2 - (\partial_1 r \partial_2 y_0 - \partial_2 r \partial_1 y_0)^2}.$$

Some limits

- Generally we could look for general solutions to the equations of motion. However, we can actually get some information using the old solution written by Kruczenski and used by Alday and Maldacena.
- One limit is where $|s|, |t|, |u| \gg r_0 \gg a_0$, and the other is where $a_0 \gg |s|, |t|, |u| \gg r_0$.
(we use a to set the scale of $|s|, |t|, |u|$ as in AM).



- In both of these limits, the action can be written as

$$S_{tot} = S_0 + \epsilon S_1 + \dots$$

with S_0 the action in pure AdS.

- To evaluate the correction to the action on the solution, one plugs in the 0th order solution into the first order action.

Example

- To be concrete, we work the example

$$a_0 \gg |s|, |t|, |u| \gg r_0$$

- For such a solution $r(y_1, y_2) \ll a_0$ and so we may expand for small $r(y_1, y_2)/a_0$.

$$S_{tot} = \frac{R^2 \sqrt{n_1}}{2\pi\alpha'} \int dy_1 dy_2 \frac{1}{r^2} \left(1 + \frac{1}{2} \frac{r^4}{a_0^4} \frac{n_2}{n_1} + \dots \right) \times \sqrt{1 - (\partial_i y_0)^2 + (\partial_i r)^2 - (\partial_1 r \partial_2 y_0 - \partial_2 r \partial_1 y_0)^2}.$$

- Note that the correction has extra powers of r in the numerator: this causes the final answer to be finite and not depend at all on r_0 .

Example

- We want to evaluate the second part on the AM solution written in y_1, y_2 coordinates

$$r = \left(\frac{\left(4y_1^2 b^2 - 2a^2 + 2a\sqrt{a^2 - 4by_1 y_2} + 4by_1 y_2 \right)}{by_1^2} \right. \\ \times \left. \frac{\left(4y_2^2 b^2 - 2a^2 + 2a\sqrt{a^2 - 4by_1 y_2} + 4by_1 y_2 \right)}{by_2^2} \right)^{1/2} \\ \times \frac{y_1 y_2}{2(a - \sqrt{a^2 - 4by_1 y_2})} \\ y_0 = \frac{1}{2b} \sqrt{(1 + b^2)} \left(a - \sqrt{a^2 - 4by_1 y_2} \right).$$

- looks messy... but there is an analytic result

Example

- The final result is

$$\Delta S = \frac{R^2 \sqrt{n_1}}{2\pi\alpha'} \frac{n_2 a^4}{6a_0^4 n_1} \frac{(1+b^2) \ln\left(\frac{(1+b)^2}{(1-b)^2}\right) - 4b}{b^3}.$$

- or written in terms of Madelstam variables s, t

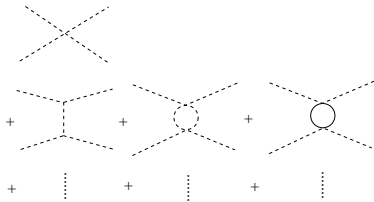
$$S_1 = \frac{R^2 \sqrt{n_1}}{2\pi\alpha'} \frac{n_2 \pi^4}{n_1 a_0^4} \left(\frac{4 t^2 s^2 (s+t)}{3 (s-t)^3} \ln\left(\frac{s}{t}\right) - \frac{8}{3} \frac{t^2 s^2}{(s-t)^2} \right)$$

- Note as $s \rightarrow t$ this is finite.
- Note also this goes as $\frac{1}{a_0^4} \sim \frac{1}{M_W^4}$

- So, what have we computed? We have

$$\begin{aligned}\mathcal{A} &= \exp(-S_{tot}) &= \exp(-S_{AM}) \exp(-S_1) \\ & &= \exp(-S_{AM}) (1 - S_1)\end{aligned}$$

- This is the contribution to the amplitude coming from massive “Ws” running in loops to leading order in $1/M_W$.



Overview

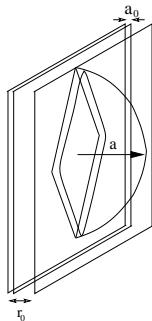
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- This suggests to expand the field theory result the same way.

$$\begin{aligned} & \mathcal{A}(s, t, u, M, \Lambda_{IR}) \\ &= \mathcal{A}(s, t, u, M = \infty, \Lambda_{IR}) \left(1 + \frac{1}{M^n} \times (\mathcal{X}) + \dots \right) \end{aligned}$$

- ... yet to be done.

- The other limit can be explored as well, namely $|s|, |t|, |u| \gg r_0 \gg a_0$



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- The $s \neq t$ case is hard:
- We cut at $r = \text{constant}$ with

$$r = \left(\frac{\left(4y_1^2 b^2 - 2a^2 + 2a\sqrt{a^2 - 4by_1 y_2} + 4by_1 y_2 \right)}{by_1^2} \right) \times \left(\frac{\left(4y_2^2 b^2 - 2a^2 + 2a\sqrt{a^2 - 4by_1 y_2} + 4by_1 y_2 \right)}{by_2^2} \right)^{1/2}$$

$$\times \frac{y_1 y_2}{2(a - \sqrt{a^2 - 4by_1 y_2})}$$

$$y_0 = \frac{1}{2b} \sqrt{(1 + b^2)} \left(a - \sqrt{a^2 - 4by_1 y_2} \right).$$

- so $r = \epsilon$ gives $y_1(y_2, \epsilon)$ in principal. In practice, the above is difficult, unless $b = 0$ i.e. $s = t$.
- We only analyze $s = t$.

- In this case, we find

$$S_0 = \frac{R^2 \sqrt{n_1 + n_2}}{2\pi\alpha'} \left[\frac{1}{2} \left(\ln \left(\frac{r_0^2}{8\pi^2(-s)} \right) \right)^2 - 2 \ln(2)^2 - \frac{1}{4} - \frac{1}{6}\pi^2 \right]$$

- and the correction to the action

$$S_1 = \frac{R^2 \sqrt{n_1 + n_2}}{2\pi\alpha'} \frac{n_1 n_2 a_0^2}{(n_1 + n_2)^2} \frac{5}{4} \left[\frac{\left(12 \ln(2) + 1 - 4 \ln \left[\frac{2r_0^2}{\pi^2(-s)} \right] \right)}{r_0^2} + \dots \right].$$

- not too much to say: similar field theory limits look promising.

Future work

- Bern, Dixon and Smirnov calculation in Coulomb phase? Expected dependence on $n_2/n_1, 1/M_W$?
- Answer the holographically? Construct the appropriate worldsheet? Interpolates between an unbroken $SU(n_1 + n_2)$ and the broken phase?

